On the Collisionless Damping and Saturation of Zonal Flows

P.H. Diamond

CMTFO, CASS and Dept. of Physics, UCSD, USA

Collaborators: M. Malkov[1], P.-C. Hsu[1], S.M. Tobias[2],A. Ashourvan[1], G. Tynan,[1]O.D. Curcan[3]

[1]*UCSD*, [2]*Leeds*, [3]*Ecole Polytechnique*

Caveats

- Time limit $\Rightarrow$ reduction in scope relative to abstract
- Works in Progress

Outline

- ▷ Zonal Flows: Some Things We Know
- ▷ Major Unresolved Issue: Damping at Low Collisionality
- ▷ Something General: Non-Perturbative Approaches to the Structure of the Reynolds Stress.
- ▷ Something Specific: A Second Look at Reynolds Work (R_{T}) and What it (really) Means.
- ▷ Something Relevant: L→ H Threshold of Low Collisionality?
- ▷ Discussion

Advertisement

" Zonal Flows and Pattern Formation"

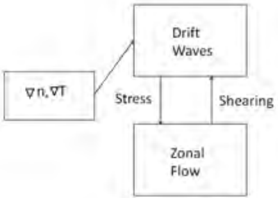
O. D. Gurcan and P.D.

J. Phys. A, in press.

- ▷ emphasized *real space* approach, in contrast to PPCF.
- ▷ enlarged discussion of non-MFE connections.

I.) Zonal Flows: Some Things We know

- sheared $n = m = 0$ $E \times B$ flows
- minimal inertia (Hasegawa)
  minimal damping (NMR)
  no radial transport



- **k**: nonlinear coupling drive: modulation, parametrics, etc. (see D I^2 2005)
- Better: *Space* (GD, 2015)
  →inhomogeneous PV mixing drives
  → PV:

$$q = \nabla^2 \psi + \beta y \qquad \text{(QG)}$$

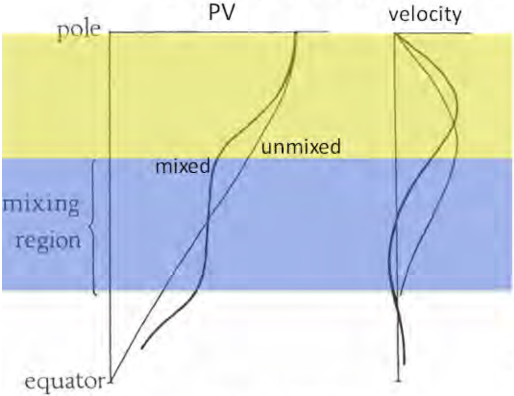
$$q = n - \nabla^2 \phi \qquad \text{(HW)}$$

$$\langle \tilde{v}_r \tilde{n} \rangle \implies \left\langle \tilde{v}_r \nabla^2 \tilde{\phi} \right\rangle \implies -\partial_r \langle \tilde{v}_r \tilde{v}_\vartheta \rangle \implies \text{Flow}$$

Inhomogeneous PV mixing

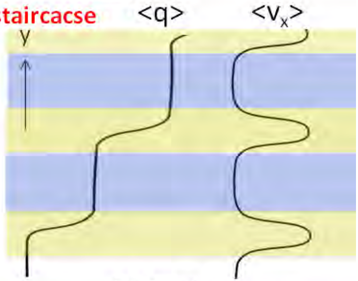
- **PV mixing is the fundamental mechanism for zonal flow formation**

$$\delta(\mathrm{PV}) \rightarrow \delta(\nabla^2\psi) \rightarrow \delta(\psi) \rightarrow \upsilon = \nabla \times \psi$$

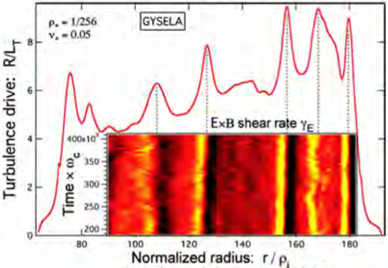


McIntyre 1982

→ PV staircacse



E x B staircase



G. Dif-Pradalier et al 2010

Predator-Prey Paradigm

- ▷ Drift Waves → Prey (P.D., et. al 1994)

Zonal Flow → Predator

$$\begin{aligned} \partial N/\partial t &= \gamma N - \alpha V^2 N - \Delta\omega N^2 + ? \\ & \qquad\qquad\qquad\qquad\quad \uparrow \\ \partial V^2/\partial t &= \alpha N V^2 - \gamma_d V^2 - \left[\gamma_{NL}\left(N, V^2\right) V^2\right] \\ & \qquad\qquad\quad \downarrow \qquad\qquad \downarrow \end{aligned}$$

drag damping $\sim \nu_{i,i}$ — NL damping (?!)

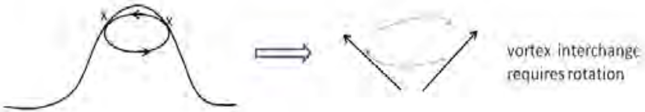
- ▷ drag regulates system! ⇒ sets fluctuation levels, etc.
- ▷ what of $\nu_{i,i} \to 0$, $\gamma_d \to 0$ limit?
- ▷ need confront nonlinear damping *and* feedback!

II.) Major Unresolved Issues: *Collisionless Damping*

- Usual rejoinder to $\nu_{i,i} \to 0$, $R/L_T \gg E/L_{Te}$?

$$\left.\begin{array}{c} -Kelvin - Helmholtz \\ \text{“}Tertiary\text{”}(Rogers) \end{array}\right\} \begin{array}{c} linear\ instability\ of \\ strongly\ sheared\ ZF. \end{array}$$

  - -∇n, ∇T_i, $\nabla V_{||}$ driven (beyond classic Rayleigh)
  - Complications:
  - magnetic shear imposes significant constraint



- linear instability $\to \nu_{eff}$??

Issues cont'd...

- numerical results controversial, inconclusive
  - KH must be accompanied by feed back to fluctuation intensity (not addressed).
  - no work on CTEM-driven ZFs

    N.B. :
  - flat -q, weak-shear "de-stiffened mode" (ala' JET) is relevant improved core confinement regime
  - $q' \to 0$ removes $\hat{s}$ constraint $\Rightarrow$ ZF stability? (c.f. Z. Lu, et. al. ; TTF 2015)
  - Question: How weak must $\hat{s}$ be?

Issues cont'd...

- More interestingly:
  - linear stability can't be only feedback mechanism in nonlinearly coupled system
  - ambient fluctuations scatter $\nabla u \Rightarrow$ effective viscosity?
    Related:
  - models perturbative - often capture only lowest order eddy tilting effects
    - non-perturbative approaches?
    - general models of vorticity flux?
  - impact of feedback on evolution?
    i.e. see what happens....

Something general:

Non-Perturbative Approaches to

the Reynolds Stress

P.-C. Hsu, P.D., S.M. Tobias, Phys. Rev. E, 2015

P.-C. Hsu, P.D., PoP 2015

Non-perturbative approaches

- PV mixing in space is essential in ZF generation.

Taylor identity: $\langle \tilde{v}_y \nabla^2 \tilde{\phi} \rangle = -\partial_y \langle \tilde{v}_y \tilde{v}_x \rangle$

vorticity flux — Reynolds force

Key:
How represent inhomogeneous PV mixing

General structure of PV flux?
→relaxation principles!

most treatment of ZF:
-- perturbation theory
-- modulational instability
(test shear + gas of waves)
~ linear theory

-> physics of evolved PV mixing?
-> something more general?

non-perturb model 1: use selective decay principle

What form must the PV flux have so as to dissipate enstrophy while conserving energy?

non-perturb model 2: use joint reflection symmetry

What form must the PV flux have so as to satisfy the joint reflection symmetry principle for PV transport/mixing?

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General principle: selective decay

- 2D turbulence conservation of energy and potential enstrophy

→ dual cascade

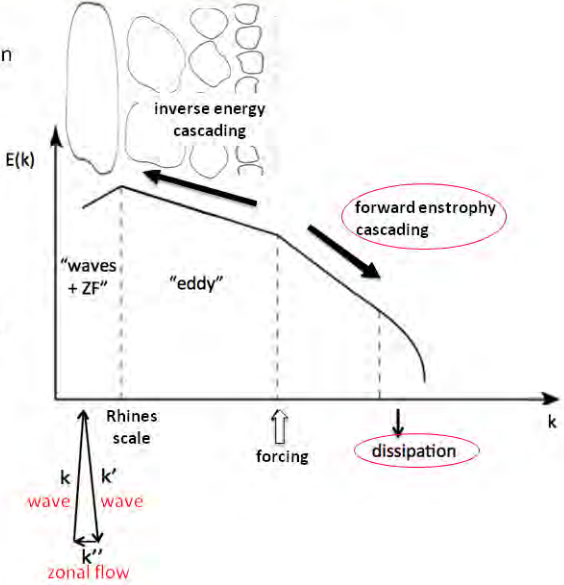
→ Minimum enstrophy state

- eddy turnover rate and Rossby wave frequency mismatch are comparable

$$\frac{\partial \omega}{\partial t} + \vec{u} \cdot \nabla \omega + \beta v = 0$$

$$\frac{U}{LT} \quad \frac{U^2}{L^2} \quad \beta U$$

→ Rhines scale $L_R \sim \sqrt{\frac{U}{\beta}}$



non-perturb model 1

Using selective decay for flux

	minimum enstrophy relaxation (Bretherton & Haidvogel 1976)	analogy ⟷	Taylor relaxation (J.B. Taylor, 1974)
turbulence	2D hydro		3D MHD
conserved quantity (constraint)	total kinetic energy		global magnetic helicity
dissipated quantity (minimized)	fluctuation potential enstrophy		magnetic energy
final state	minimum enstrophy state flow structure emergent		Taylor state force free B field configuration
structural approach	$\frac{\partial}{\partial t}\Omega<0 \Rightarrow \Gamma_E \Rightarrow \Gamma_q$		$\frac{\partial}{\partial t}E_M<0 \Rightarrow \Gamma_H$

dual cascade (conserved quantity / dissipated quantity)

- flux? what can be said about dynamics?

→ structural approach (this work): *What form must the PV flux have so as to dissipate enstrophy while conserving energy?*

General principle based on general physical ideas → useful for dynamical model

non-perturb model 1

PV flux

→ PV conservation

mean field PV: $$\frac{\partial\langle q\rangle}{\partial t}+\partial_y\underbrace{\langle v_y q\rangle}_{\Gamma_q}=\nu_0\partial_y^2\langle q\rangle$$

Γ_q : mean field PV flux

Key Point: what form does PV flux have s/t dissipate enstrophy, conserve energy

selective decay

→ energy conserved $E=\int\frac{\left(\partial_y\langle\phi\rangle\right)^2}{2}$

$$\frac{\partial E}{\partial t}=\int\langle\phi\rangle\partial_y\Gamma_q=-\int\partial_y\langle\phi\rangle\Gamma_q \qquad \Rightarrow\Gamma_q=\frac{\partial_y\Gamma_E}{\partial_y\langle\phi\rangle}$$

→ enstrophy minimized $\Omega=\int\frac{\langle q\rangle^2}{2}$

$$\frac{\partial\Omega}{\partial t}=-\int\langle q\rangle\partial_y\Gamma_q=-\int\partial_y\left(\frac{\partial_y\langle q\rangle}{\partial_y\langle\phi\rangle}\right)\Gamma_E$$

$$\frac{\partial\Omega}{\partial t}<0\Rightarrow\Gamma_E=\mu\partial_y\left(\frac{\partial_y\langle q\rangle}{\partial_y\langle\phi\rangle}\right) \qquad \Rightarrow\Gamma_q=\frac{1}{\partial_y\langle\phi\rangle}\partial_y\left[\mu\partial_y\left(\frac{\partial_y\langle q\rangle}{\partial_y\langle\phi\rangle}\right)\right]$$

general form of PV flux

μ: parameter TBD

$\partial_y\langle\phi\rangle \to \langle v_x\rangle$

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Structure of PV flux

$$\Gamma_q = \frac{1}{\langle v_x \rangle}\partial_y\left[\mu\,\partial_y\left(\frac{\partial_y\langle q\rangle}{\langle v_x\rangle}\right)\right] = \frac{1}{\langle v_x \rangle}\partial_y\left[\mu\left(\underbrace{\frac{\langle q\rangle\partial_y\langle q\rangle}{\langle v_x\rangle^2}}+\underbrace{\frac{\partial_y^2\langle q\rangle}{\langle v_x\rangle}}\right)\right]$$

μ: diffusion parameter calculated by perturbation theory, numerics...

diffusion and hyper diffusion of PV

<--> usual story : Fick's diffusion

relaxed state:

Homogenization of $\frac{\partial_y\langle q\rangle}{\langle v_x\rangle}$ → allows staircase

characteristic scale $\ell_c \equiv \sqrt{\left|\frac{\langle v_x\rangle}{\partial_y\langle q\rangle}\right|}$

$\ell > \ell_c$: zonal flow growth

$\ell < \ell_c$: zonal flow damping (hyper viscosity-dominated)

⟺

Rhines scale $L_R \sim \sqrt{\frac{U}{\beta}}$

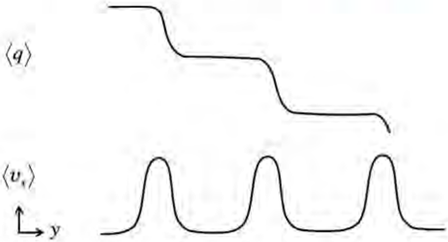
$\ell > L_R$: wave-dominated

$\ell < L_R$: eddy-dominated

PV staircase

relaxed state: homogenization of $\frac{\partial_y \langle q \rangle}{\langle v_x \rangle}$ → PV gradient large where zonal flow large

→ Zonal flows track the PV gradient → PV staircase



- Highly structured profile of the staircase is reconciled with the homogenization or mixing process required to produce it.
- Staircase may arise naturally as a consequence of minimum enstrophy relaxation.

What sets the "minimum enstrophy"

- Decay drives relaxation. The relaxation rate can be derived by linear perturbation theory about the minimum enstrophy state

$$\langle q \rangle = q_m(y) + \delta q(y,t)$$
$$\langle \phi \rangle = \phi_m(y) + \delta\phi(y,t)$$
$$\partial_y q_m = \lambda \partial_y \phi_m$$
$$\delta q(y,t) = \delta q_0 \exp(-\gamma_{rel} t - i\omega t + iky)$$

$$\gamma_{rel} = \mu\left(\frac{k^4 + 4\lambda k^2 + 3\lambda^2}{\langle v_x \rangle^2} - \frac{8q_m^2(k^2+\lambda)}{\langle v_x \rangle^4}\right)$$
$$\omega_k = \mu\left(-\frac{4q_m k^3 + 10 q_m k\lambda}{\langle v_x \rangle^3} - \frac{8q_m^3 k}{\langle v_x \rangle^5}\right)$$

- The condition of relaxation (modes are damped):

$\Upsilon_{rel} > 0 \rightarrow k^2 > \frac{8q_m^2}{\langle v_x \rangle^2} - 3\lambda$

$k^2 > 0 \rightarrow \frac{8q_m^2}{\langle v_x \rangle^2} > 3\lambda$ → Relates q_m^2 with ZF and scale factor

-- ZF cannot grow arbitrarily large, and is constrained by the enstrophy

-- To sustain a zonal flow in the minimum enstrophy state, a critical residual enstrophy density is needed.

→ q_m^2: the 'minimum enstrophy' of relaxation, related to scale

Something Relevant:

L$\rightarrow$H Threshold at Low Collisionality

M. Malkov, P.D., et. al.; PoP 2015

P.-C. Hsu, P.D., S.M. Tobias, Phys. Rev. E, 2015

M.M., P.D., et. al. TTF 2015 (Invited)

Emerging Scenario for L→H

LH-triggering sequence of events

$Q \uparrow \implies \tilde{n},\ \tilde{v} \uparrow \implies <\tilde{v}_r \tilde{v}_\vartheta>; \quad <\tilde{v}_r \tilde{v}_\vartheta> d <v>/\mathrm{dr} \uparrow \implies |\tilde{n}|^2 \downarrow$, etc.

$\implies \nabla P_i| \uparrow \implies$ lock in transition (*Tynan et al. 2013*)

- ∇T etc. drives turbulence that generates low frequency shear flow via Reynolds stress
- Reynolds work coupling collapses the turbulence thus reducing particle and heat transport
- Transport weakens $\rightarrow \nabla \langle P_i \rangle$ builds up at the edge, accompanied by electric field shear $\nabla \langle P_i \rangle \rightarrow \langle V_E \rangle'$
- locks in $L \rightarrow H$ transition: (*see Hinton ,Staebler 1991, 93*)
- Complex sequence of Transition Evolution and Alternative End States (I-mode) possible (*D. Whyte et al. 2011*)

Some Questions:

Nucl. Fusion 53 (2013) 113003

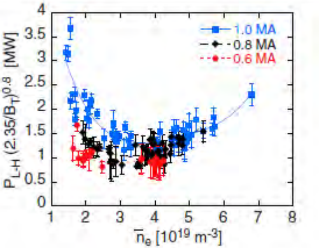
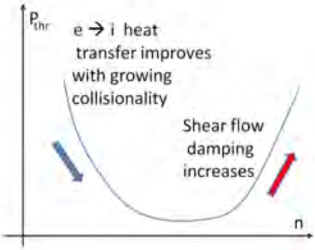


Figure 3. Power threshold versus density for the L–H transition normalized to $|B_T| = 2.35$ T by the $B_T^{0.8}$ dependence. The fits to the P_{L-H} data indicated here are also shown in figure 4. The error bars include all the contributions to P_{loss}. The larger error bars are due to the dW/dt term for discharges with a rather strong change of heating power before the occurrence of the L–H transition.

Ryter et al 2013

- How does the scenario relate to the Power Threshold?
  - Is $P_{thr}(n)$ minimum recoverable?
- Micro-Macro connection in threshold, if any?
- How does micro-physics determine threshold scalings?
- What is the physics/origin of $P_{thr}(n)$? Energy coupling?
- Will P_{min} persist in collisionless, electron-heated regimes (ITER)?

Scenario (inspired partly by *F. Ryter, 2013-14*)



- $\nabla P_i|_{\text{edge}}$ essential to 'lock in' transition
- to form ∇P_i at low n, etc. need (collisional) energy transfer from electrons to ions

$$\frac{\partial T_e}{\partial t} + \frac{1}{r}\frac{\partial}{\partial r} r\Gamma_e = -\frac{2m}{M\tau}(T_e - T_i) + Q_e$$

$$\frac{\partial T_i}{\partial t} + \frac{1}{r}\frac{\partial}{\partial r} r\Gamma_i = +\frac{2m}{M\tau}(T_e - T_i) + Q_i$$

- suggests that the minimum is due to:
  - P_{thr} decreases due to increasing heat transfer from electrons to ions
  - P_{thr} increases (stronger edge∇P_i driver needed) due to increase in shear flow damping
  - Power and edge heat flux are not the only crit. variables: also need the ratio of electron energy conf. time to exceed that of $e - i$ temp. equilibration $T_r = \tau_{Ee}/\tau_{ei}$- most important in pure e-heating regimes
    - $T_r \gg 1$ somewhat equivalent to direct ion heating
    - $T_r \ll 1$ ions remain cold → no LH transition (or else, it's anomalous!)

Predator-Prey Model Equations

- Based on 1-D numerical 5-field model (*Miki & Diamond++ 2012,13+*)
- Currently operates on 6 fields ($+P_e$) with self-consistenly evolved transport coefficients, anomalous heat exchange and NL flow dissipation (*MM, PD, K. Miki, J. Rice and G. Tynan, PoP 2015*)
- Heat transport, + Two species, with coupling, i,e (*anomalous heat exchange* in color):

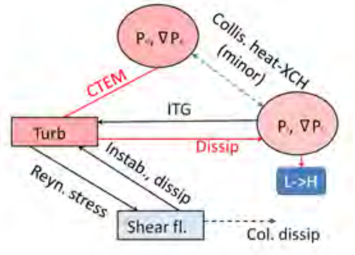
$$\frac{\partial P_e}{\partial t} + \frac{1}{r}\frac{\partial}{\partial r} r\Gamma_e = -\frac{2m}{M\tau}(P_e - P_i) + Q_e - \gamma_{CTEM} \cdot I$$

$$\frac{\partial P_i}{\partial t} + \frac{1}{r}\frac{\partial}{\partial r} r\Gamma_i = \frac{2m}{M\tau}(P_e - P_i) + Q_i + \gamma_{CTEM} \cdot I + \gamma_{ZFdiss} \cdot I$$

$$\Gamma = -(\chi_{neo} + \chi_t)\frac{\partial P}{\partial r}, \quad \gamma_{ZFdiss} = \gamma_{visc}\left(\frac{\partial\sqrt{E_0}}{\partial r}\right)^2 + \gamma_{Hvisc}\left(\frac{\partial^2\sqrt{E_0}}{\partial r^2}\right)^2$$

- I and E_0 - DW and ZF energy (next VG), plasma density and the mean flow, as before

Equations cont'd; Anomalous Heat Exchange



- in high T_e low n regimes (pure e-heating) the thermal coupling is anomalous (through turbulence)
- ZF dissip. (KH?) supplies energy to ions, and returns energy to turbulence
- DW turbulence:

$$\frac{\partial I}{\partial t} = \left(\gamma - \Delta\omega I - \alpha_0 E_0 - \alpha_V \langle V_E \rangle'^2\right) I + \chi_N \frac{\partial}{\partial r} I \frac{\partial I}{\partial r}, \quad \chi_N \sim \omega_* C_s^2$$

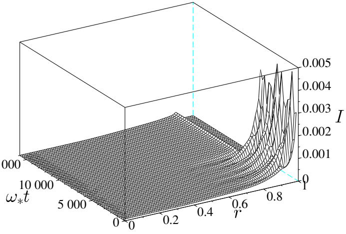
Driver : $\gamma = \gamma_{ITG} + \gamma_{CTEM} + \mathit{NL\ ZF\ Dissip\ less\ P_i\ Heat\ (currently\ balanced)}$

- ZF energy:

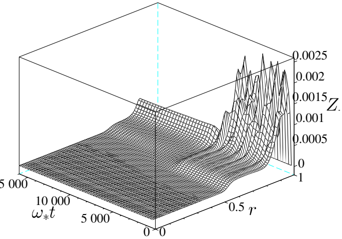
$$\frac{\partial E_0}{\partial t} = \left(\frac{\alpha_0 I}{1 + \zeta_0 \langle V_E \rangle'^2} - \gamma_{damp}\right) E_0, \quad \gamma_{damp} = \gamma_{col} + \gamma_{ZFdiss} \cdot I/E_0$$

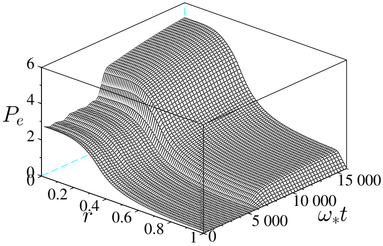
$\gamma_{ZFdiss} = \gamma_{visc} \left(\frac{\partial\sqrt{E_0}}{\partial r}\right)^2 + \gamma_{Hvisc} \left(\frac{\partial^2\sqrt{E_0}}{\partial r^2}\right)^2$ - toy model form (work in progress)

Model studies: Transition (Collisional Coupling)



- ion heat dominated transition $H_{i/(i+e)} = 0.7$
- strong pre-transition fluctuations of all quantities
- well organized post-transition flow
- strong P_e edge barrier





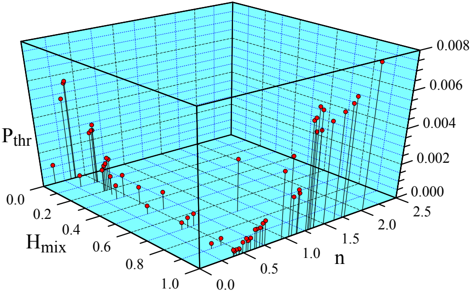
Model Studies: Control Parameters

- Heating mix

$$H_{i/(i+e)} \equiv \frac{Q_i}{Q_i + Q_e} \quad (\text{aka } H_{\text{mix}})$$

- Density (center-line averaged) is NOT a control parameter. It is measured at each transition point
- Related control parameter is the reference density given through BC and fueling rate
- There is a complicated relation between density and ref. density
- Other control parameters:
  - fueling depth
  - heat deposition depth and width, etc.
    $\rightarrow$they appear less critical than $H_{i/(i+e)}$

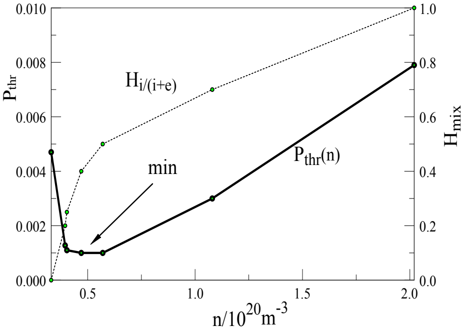
$P_{th}\left(n, H_{i/(i+e)}\right)$ scans: Recovering the Minimum



$P_{\text{thr}}\left(H_{i/(i+e)}, n\right)$-

- ▷ electron heating at lower densities
- ▷ ion heating at higher densities

- ▷ Relate $H_{i/(i+e)}$ and n by a monotonic $H_{i/(i+e)}(n)$



- ▷ $P_{\text{thr}}(n)$ min recovered!

Anomalous Regime (Preliminary)

- Anomalous Regime: $\nu_{ei} n \left(T_e - T_i \right) < \gamma_{\text{anom-ei}coupl} \cdot I$ *(Manheimer, '78; Zhao, PD, 2012; Garbet, 2013)*
  - Anomalous regime, strong electron heating (ITER)
  - n scaling coupling $\Longrightarrow$ Anomalous coupling

$$\frac{\partial T_e}{\partial t} + \frac{1}{r}\frac{\partial}{\partial r} r\Gamma_e = -\frac{2m}{M\tau}\left(T_e - T_i \right) + Q_e$$

$$\frac{\partial T_i}{\partial t} + \frac{1}{r}\frac{\partial}{\partial r} r\Gamma_i = +\frac{2m}{M\tau}\left(T_e - T_i \right) + Q_i$$

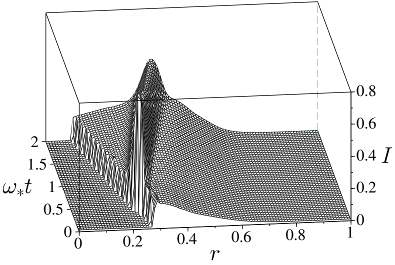
  - Anomalous coupling dominates
    - scaling + intensity dependence $\Longrightarrow$coupling

$$\frac{\partial T_e}{\partial t} + \frac{1}{r}\frac{\partial}{\partial r} r\Gamma_e = Q_e + \langle E \cdot \mathbf{J_e} \rangle \rightarrow (< 0)$$

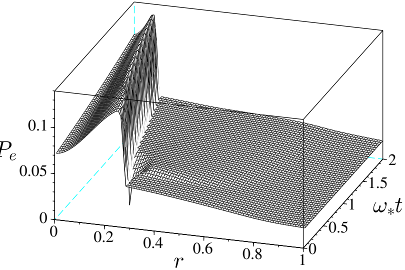
$$\frac{\partial T_i}{\partial t} + \frac{1}{r}\frac{\partial}{\partial r} r\Gamma_i = Q_e + \langle \mathbf{E} \cdot \mathbf{J_i} \rangle \rightarrow (> 0)$$

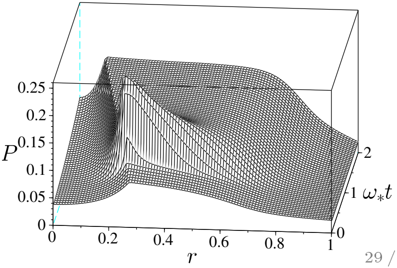
LH transition: Anomalous Transfer Dominates

Extreme limit to illustrate temperature relaxation: Pure electron heating, $\nu_{ei} \to 0$



- ▷ CTEM → Heat Exch ↗ turbulence ↘ ions
  - – Is $P_{\rm thr}$ set only by local properties at the edge?
- ▷ $e - i$ *-temperature equilibration front*
- ▷ $P_i \uparrow$ globally → strong ∇P_i at the edge → LH transition





Anomalous Regime: Issues

- An Issue:
  - Predator-Prey ⇒ Shear Flow Damping
  - ⇒Anomalous regime: collisional drag problematic
  - Low collisionality → what controls heat exchange?
  - NL damping ⇔ mediated by ZF instability (i.e. KH, tertiary; *Rogers et al 2000; Kim, PD, 2003*)
    ⇒ hyperviscosity, intensity dependent
  - Returns ZF energy to turbulence → P_i

Results so far

- transition with anomalous heat exchange happens!
- requirements for LH transition in high T_e regimes when the collisional heat exchange is weak:
  - efficient ion heating by CTEM turbulence
  - energy return to turbulence by ZF damping (caused by KH instability?!)
  - may be related to *Ryter 2014*. Subcritical $\nabla T_e \uparrow$ states at ultra-low density

Fundamental Problems Identified:

- ZF stability and saturation in CTEM regimes
- General theory of Reynolds work.